WORLD MARITIME UNIVERSITY

Malmö, Sweden

ANALYSIS OF THE OPTIMISATION OF BERTH ALLOCATION

Berth allocation with an external terminal facility

By

KUMARA H.J.K.U.
Sri Lanka

A dissertation submitted to the World Maritime University in partial
Fulfilment of the requirement for the award of the degree of

MASTER OF SCIENCE
in
MARITIME AFFAIRS

(PORT MANAGEMENT)

2003

DECLARATION

I certify that all the material in this dissertation that is not my own work has been identified, and that no material is included for which a degree has previously been conferred on me.

The contents of this dissertation reflect my own personal views, and are not necessarily endorsed by the University.

(Signature):..…………………….

(Date): ……………………….

Supervised by: Professor Akio Imai
World maritime University.

Assessor: Professor B.L.Francou
Institution: World Maritime University.

Co-assessor: Assistant Professor Stratos Papadimitriou
Institution: University of Piraeus.

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my deepest gratitude and sincere thanks to all those who were behind this task, directly and indirectly involved, providing me with the encouragement and assistance necessary to complete this project successfully.

I owe my appreciation, first and foremost, to

- The management of the Sri Lanka Ports Authority (SLPA) for nominating me for this valuable opportunity and putting their faith on me to complete this course of study.
- World Maritime University for providing me with a fellowship to study at this university.

Furthermore, I am sincerely grateful to my supervisor Professor Imai, whose research work had already inspired me, for taking a considerable period out of his valuable time to guide and assist me in the right direction even after office hours, and Professor Francou for being my assessor, and also Assistant Professor Papadimitriou for being my co-assessor.

I would also like to extend my gratitude to the entire teaching staff that taught me from the very first semester in this university to come up with such an exertion of research, and wonderful staff of the WMU library, especially Susan Wangeci-Eklow for facilitating me with all the necessary books and the assistance provided in finding materials on time.

Finally, I am deeply indebted to the support and encouragement given to me by my friends at WMU, and colleagues at SLPA, who provided me with necessary information when required and my parents and wife who took great pains during my lengthy stay away from home enabling me to make this event a reality.

ABSTRACT

Title of Dissertation: **Analysis of the optimisation of berth allocation : Berth allocation with an external terminal facility**

Degree: **MSc**

This study focuses on an efficient scheduling method, which may be applicable to berth allocation to calling vessels at the port of Colombo, Sri Lanka where there are two container terminals operating opposite to each other. Due to various reasons, some vessels calling at one of the terminals called Jaya Container Terminal (JCT) have to be moved to an external terminal facility known as South Asia Gateway Terminal (SAGT), whenever there is a congested situation at JCT. Vessels would be moved depending on their predictable waiting times for berths at JCT.

The dissertation aims at finding a methodology to develop a computer-based programme that could produce the optimum results in order to minimise the number of vessels allocated to the external terminal with resulting huge costs born by JCT in such an operation.

Based on actual statistics on arrival times, handling times and completion times of vessels, some results have been analysed in order to see how this methodology can be applied in real situations such as the case under consideration.

By means of the outcomes from the analysed data, a two-stage method of assigning berths at the JCT is proposed whilst maintaining span for the fine-tuning of the berth allocation with the minimum possible number of vessels directed to the external terminal.

The computational results proves that the methodology developed in this study is efficient enough to facilitate the daily berth allocation tasks in any container terminal with the same situation as the one discussed in this dissertation.

Key words: Berth allocation, Container terminal, External terminal, Mathematical programming, Scheduling, Waiting time.

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LIST OF ABBREVIATIONS

BAP	Berth Allocation Problem
BOT	Build, Operate and Transfer
DEBAP	Dynamic Berth Allocation with an External Terminal facility
FCFS	First-Come-First-Served
JCT	Jaya Container Terminal
MUT	Multi User Terminal
QCT	Queen Elisabeth Container Terminal
SAGT	South Asia Gateway Terminal
SEBAP	Static Berth Allocation with an External Terminal facility

CHAPTER ONE

INTRODUCTION

Many container ports in the world serve as *Dedicated Terminals* to some shipping lines, since "time means money" especially for the shipping lines. It is said, "shipping lines can not accept to loose in ports the time and money they have gained at sea" (Francou, 2002, p. 9) This has been one of the reasons for shipping lines to invest in container terminals to operate as private or dedicated terminals.

On the other hand, there are container terminals, operated as *Common User Terminals* or also known as *Multi User Container Terminals*, which are mostly state-owned. Most of these terminals use the "First Come First Served" criterion for their day-to-day operations. These terminals can be defined to be with long berths, which are capable of serving more than one ship at a time, depending on the size of the terminal facilities. Moreover, ships would be allocated to specific berths, taking into account the arrival times of the vessels, so that a ship may not always be assigned to a specific berth every time it calls.

As liner shipping is characterised with, fixed sailing schedules, and fixed ports of call by named vessels (Ma, 2002, p. 45) with minimisation of the port-to-port transit time, the productivity of a container terminal plays a major role in the entire container liner shipping. While this productivity can be measured in the box-handling rate per hour, the ship's waiting time for a berth also has a significant effect on the total time (or turn-around time) spent on a port of call. The productivity is also defined by the handling time as a reverse measurement, when the "turn-around time" of a vessel is defined to be the total of both waiting time and the handling time.

In this light, major container carriers, which operate between furthermost corners of the main shipping routes, for example Far-East and Europe, opt to have a system called Hub and Spoke system, which is one of the systems of selecting a port of call. This aims at minimising the turn-around time in port and reducing the number of port calls as far as possible, in compliance with the stipulated sailing schedules. At this juncture, the waiting time for a berth could affect adversely the tight schedules of shipping lines.

All the ports of call by such vessels should strive hard to enhance the time limitations of the shipping schedule. From this point of view, an effective principle is highly needed for allocation of berths in a container terminal.

This study is inspired with a sophisticated decision-making on the berth allocation to calling ships in one of the container terminals in the port of Colombo, Sri Lanka, where the port infrastructure is insufficient to meet increasing traffic of container ships.

Due to the strategic and ideal location in the Indian Ocean, for container traffic between the East and South East Asian regions and Europe, the port of Colombo, which is one of the major hub ports, features a fully pledged modern container terminal called Jaya Container Terminal (JCT) and another container terminal known as Queen Elisabeth Container Terminal (QCT). The latter has been operating since 1980, while the former has been in operation with four berths since 1996. The berths of JCT were put in operation one by one: the first berth in 1985, the second in 1987, the third in 1995 and finally the fourth in 1996.

Since then container-handling operations in the port of Colombo were in a monopolistic situation, attracting cargo from all around the island of Sri Lanka. Colombo Port is well connected with a huge network of feedering vessels to and from countries like India (including both East coast and West coast of India), Pakistan, Maldives, and Bangladesh etc. Due to the ever-increasing container traffic in the port of Colombo, both terminals were congested almost all the time. Thus, in order to expand the capacity of QCT to cope with the future growth of the container traffic, the government of Sri Lanka signed a concession agreement with the private sector for the project termed –"Build, Operate and Transfer (BOT) the QCT"– as an alternative terminal, which is called "South Asia Gateway Terminal (SAGT)". The arrangement of these two terminals, JCT and SAGT, are shown in Figure 1.

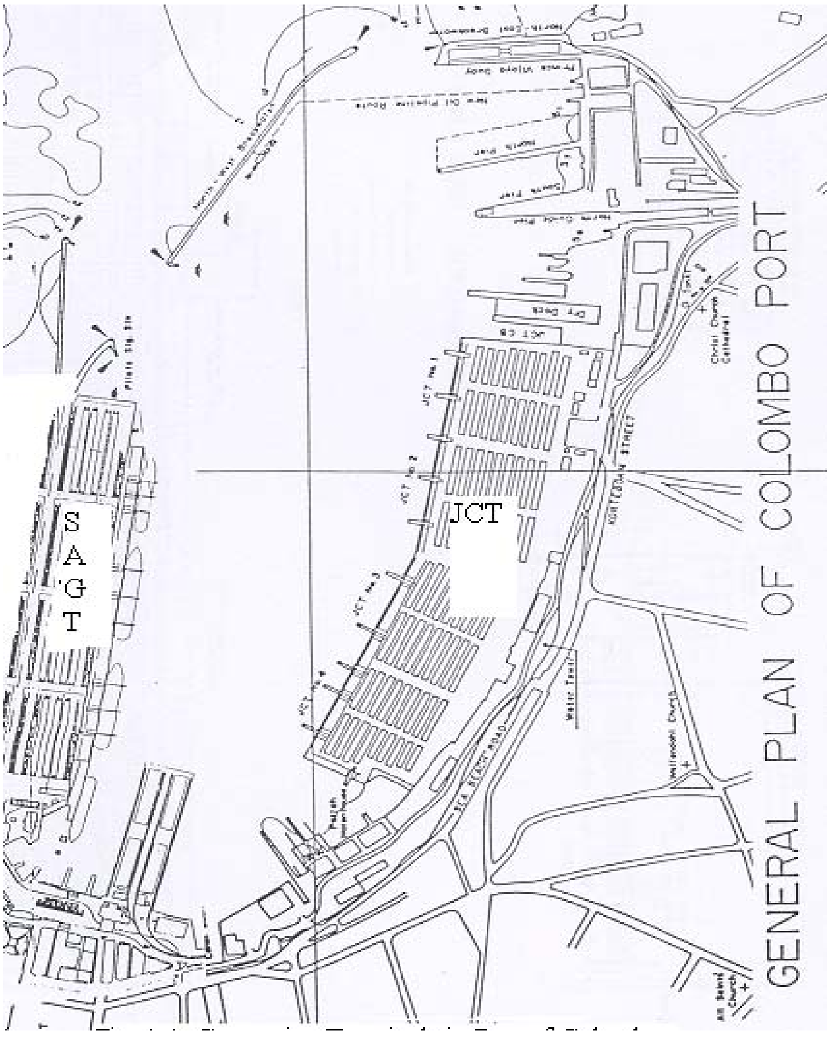

Figure 1. Competing Terminals in port of Colombo.

Source: Sri Lanka Ports Authority

With the constructions and suitable modifications, SAGT started its operations simultaneously with the development of new terminal activities. As this has been a private entity, the state-owned JCT gradually started loosing its prestigious monopolistic situation. Some feeder vessels, which were suffering from long waiting time, for berths of JCT because of the priorities given to main liner vessels, started calling at SAGT. Apparently, this resulted in loosing some of the JCT's customers, even though they were feeder vessels. Most of these containers handled at JCT are transhipment ones, as 70% of Colombo's throughput is transhipment. Unfortunately such a loss is not the only whole scenario.

In a congested situation at JCT, one has to direct the JCT calling or scheduled vessels, which may be expected to suffer from a long waiting time, to SAGT. Though this exercise may be advantageous for shipping lines, in terms of waiting time, it is not at all for JCT. Such an exercise results in collection of various extra cost elements involved, other than loosing a customer itself. To understand the whole scenario clearly, the following facts should be taken into account.

A vessel is calling at JCT either to unload or load containers or both at JCT.

1. Unloading containers.
    - These containers are due either to be delivered as local containers from JCT or to be reloaded as transhipment containers to ships that are due at a latter stage and therefore they have to be stacked in the JCT yard.
2. Loading containers at JCT.
    - These containers are planned and ready to be loaded to a vessel, which is berthed at JCT and they are already lying in the JCT yard.

When a vessel with operations, discussed above, is berthed at the external terminal (SAGT), JCT has to

- Haul containers to/from SAGT yard from/to JCT yard.

- Perform all the necessary mounting and demounting of containers in the yard in order to send/stack containers in the JCT yard.
- Perform shifting of containers, in the stacks, if necessary.

All these activities involve huge costs in terms of money and time. All the relevant costs have to be born by JCT alone.

When all these scenarios are taken into consideration, an underlying factor to be considered is related to the waiting times of vessels for berths. The operator makes an effort to minimize ship's waiting time by scheduling berth allocation efficiently. In addition, if the expected waiting time of a ship exceeds the criterion, the ship is directed to the external terminal to meet its promised turn-around time. As mentioned before, such re-assignment of ships to the external terminal imposes additional costs; therefore, scientific methods should be applied for the efficient berth allocation scheduling. Some authors have already addressed the berth allocation problem (BAP), minimising the waiting times for berths in order to have a quick turn-around time of a vessel.

This dissertation discusses the issue of berth allocations for calling vessels, focusing on a theoretical approach of the minimization of the number of vessels allocated to the external terminal, to enhance a quick turn-around time.

The dissertation has a literature review regarding the planning of berth allocation in the next chapter. Chapter three addresses the problem formulation while a solution algorithm is introduced in chapter four. Computational experiments are performed in chapter five followed by chapter six, which introduces possible further improvements in the future to the problem and chapter seven concludes the dissertation.

CHAPTER TWO

LITERATURE REVIEW

Studies have been done regarding efficient operations and berth scheduling in ports by various authors. Some studies treat the optimal berth allocation. Among those, a typical objective is to minimise the turn-around time. Most of the studies are on terminals, which are privately operated by specific shipping lines, while very few studies deal with multi-user terminal (MUT) systems.

Kao and Lee (1996), in a study of dock arrangement and ship discharging, treated the berth allocation with the minimisation of turn-around time by using an interactive procedure, which passes the necessary information between the dock arrangements and ship discharging. This is not the exact foundation for this area of study.

Motivated by more efficient berth usage in the HIT terminal in Hong Kong, Lai and Shih (1992) proposed a heuristic algorithm for berth allocation based on a First-Come-First-Served (FCFS) strategy.

Berth planning in naval ports and commercial ports has a major difference. In naval ports, a vessel that is already being served may be shifted due to an arrival of another vessel to be served. This is highly unlikely in commercial ports. Brown, Lawphongpanich and Thurman (1994) and Brown, Cormican and Lawphongpanich (1997) studied the ship handling in naval ports. Maximising the sum of benefits for ships, while in port, an optimal set of ship-to-berth assignment is identified in their study.

When allocating berths to container vessels in an MUT, in general, there may be some dissatisfaction about the service order of ships. Imai, Nagaiwa and Tat (1997) discuss berth allocation problem that minimises the sum of turn-around time of ships and that minimises dissatisfaction of the ships in terms the berthing order. The problem assumes a static situation where all incoming ships have arrived for planning before a given time horizon. A heuristic procedure was developed to obtain a good solution with the minimum amount of computation without restrictions regarding the ship's draft or length.

In reality, when allocating berths in an MUT, ships call at a given port more or less randomly. That means, even though there are specified calling schedules of vessels, berths fall vacant at different times. This causes ships to call at different times. Imai, Nishimura and Papadimitriou (2001), addressed the problem of determining a dynamic berth assignment, where some ships arrive at the port during the planning horizon, in an MUT without the ship-related physical restrictions. In that effort, a heuristic procedure based on the Lagrangian relaxation of the original problem was developed. Furthermore, a large amount of computational experiments showed that the proposed algorithm was adaptable to real world applications.

For the same end, Nishimura, Imai and Papadimitriou (2001), extended the dynamic berth allocation problem to develop a heuristic procedure based on the genetic algorithm, taking into account water depth and quay length restrictions.

Major container hub ports such as Hong Kong, Pusan, Hamburg and Rotterdam use the MUT concept to reduce redundant terminal space for substantial cost savings in cargo handling tasks. The past berth allocation problems for the MUT, treat each vessel equally. Imai, Nishimura and Papadimitriou (2003) modified the existing formulation of berth allocation problem in order to treat calling vessels with various service priorities. Such a treatment is necessary because some vessel operators desire high priority over others due to various factors such as meeting time limitations in order to be able to ply with the scheduled convoys at canals. Modifications of the existing berth allocation formulation were applied and the solution is obtained by a genetic algorithm-based heuristic.

Discrete and continuous locations are the typical schemes of berth allocation. The former has the advantage of easy planning and the disadvantage of weakness in efficient terminal usage, while the latter is more appropriate in terms of flexibility in berth allocation in busy hub ports where various sizes of ships are calling. All the aforementioned studies assume the discrete location, while the followings are based on the continuous location.

Guan, Xiao, Cheung and Li (2002) examine a scheduling problem where some vessels require simultaneous operations of multiple consecutive quay cranes. They developed a heuristic for the problem and have performed worst-case analysis. This problem was motivated by the operation of berth allocation. Their objective is to minimise the total weighted completion time of vessels.

Park and Kim (2003) discuss a method for scheduling berths and quay cranes. Considering various practical constraints, an integer-programming model was formulated in order to solve the mathematical model. A solution is obtained in two-phases where the number of cranes assigned to each vessel at each time segment is found, determining berthing position and time. The sub-gradient optimisation technique is applied in the first phase, while the dynamic programming technique is applied in the second phase based on the solution found in the first phase.

Also Park and Kim (2002) formulated a mixed integer programme for berth scheduling problem. To ease the computational part of the mixed integer programme, the formulation is converted into another integer linear programme in which the solution space of the berth and time is discretised. Using a sub-gradient optimisation technique, a Lagrangean relaxation model of the discretised model is solved.

In Kim and Moon (2003), a mixed-integer-linear-programming (MIP) model was formulated for the berth-scheduling problem. To find a near optimal solution, a simulated annealing algorithm is applied to the berth-scheduling problem. Experimental results show that solutions obtained from the simulated annealing algorithm are close to the optimal solution found by the MIP.

CHAPTER THREE

PROBLEM FORMULATION

3.1) Introduction:

There are various parameters, which measure port productivities. Among these, with regard to container vessels, box-handling rates and waiting times for berthing are the main benchmarks.

The main concern, in this study, is the minimisation of the waiting time. Handling times of vessels vary depending on the allocation of a berth to a vessel because movements of containers to and from the container yard differ according to the stacking locations of the containers.

As mentioned in the previous chapter, taking into account this fact, Imai et al. (1997) propose a static version of the berth allocation problem (BAP) where all the ships have arrived before a given planning horizon. Extending this criterion, Imai et al. (2001) studied the dynamic version of the BAP, where ships can arrive while work is in progress.

This study, based on the works of the above-mentioned authors, develops a BAP with an External Terminal facility, and for easy reference the Static Berth Allocation with an External Terminal and the Dynamic Berth Allocation with an External Terminal are referred to as *SEBAP* and *DEBAP,* respectively.

According to the definition of the problem, the terminal operator defines a maximum waiting time limit for all calling vessels. Whenever there is a violation of the stipulated time limit, then the violating vessel is directed to the external terminal. Since this is an operation in which huge costs are involved, the terminal operator tries to minimise the number of vessels, which should be directed to the external terminal. It is assumed that there are no waiting times imposed to the directed ships for the external terminal in this exercise. One way of minimising vessels allocated to the external terminal is that to stipulate a long waiting time for the ships. But this will negatively affect the quality of the services of the terminal, apparently resulting in a long turn-around time of vessels. Therefore, the objective of the terminal is to analyse this situation, keeping intact the total service time to the satisfaction of the calling vessels and ways of minimising the costs related to allocation of vessels to the external terminal by changing the stipulated waiting time limit criterion.

First, SEBAP is analysed in order to see how the solution, in this regard, works and then DEBAP is analysed.

3.2) Formulation of SEBAP:

Assumptions made to analyse SEBAP are:

- There are no constraints such as ships' draft and length
- All the ships can be served at any berth.
- All the ships have arrived before a given planning horizon.
- The handling time of each ship depends on the berth in which it is served.
- Handling times of each ship at each berth are known.

It is considered that the MUT is comprised of a long wharf. In practise different ships, having different lengths, are allocated to specific berths depending on the lengths of the berths. For the simplicity in the solution of the problem, it is assumed that the restrictions regarding the lengths and the drafts, mentioned above, do not exist. The berths are represented by several blocks where each assignment of berthing of vessels is shown as a rectangular box in a Gantt chart, which is illustrated in Figure 2. Ships would be served according to the rectangular assignments at respective berths in Figure 2. Ship 3 is served as the first vessel at berth 1, and then ship 1 as the second ship and ship 10 as the third ship. At berth 2, ship 2 as the first ship, ship 7 as second and so on.

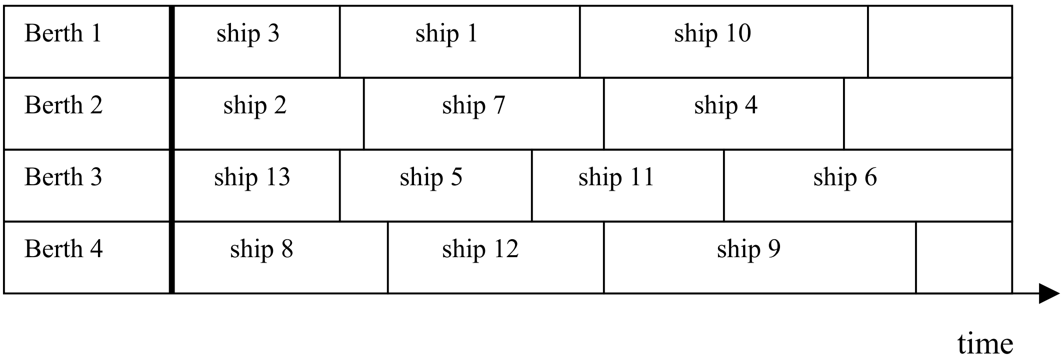

Figure 2. Allocation of berths.

The following notations are used in the formulation.

$i(=1,....,Q) \in B$: set of berths where Q is the external terminal

$j(=1,....,T) \in V$: set of ships

$k(=1,....,T) \in U$: set of service orders

A_j : arrival time of ship j

P_k : subset of U such that $P_k = \{p | p < k \in U\}$

S_i : time when berth i becomes idle for the planning horizon

C_{ij} : handling time spent by ship j at berth i

x_{ijk} : $\begin{cases} 1, \text{if ship } j \text{ is serviced as the } k\text{th ship at berth } i \\ 0, \text{otherwise} \end{cases}$

where x_{ijk} ‘s are decision variables.

According to the assumption all the ships have arrived before a given planning horizon,

$$S_i \geq A_j \text{ for all } i \text{ and } j.$$

For example, the situation at the berth 3 in Figure 2 can be analysed using a Gantt chart. Ships 13, 5, 11 and 6 have been assigned to berth 3 in the order of the service. Therefore, ship 13 is served first at berth 3 and then 5, and thereafter 11 and finally ship 6, according to that particular planning horizon.

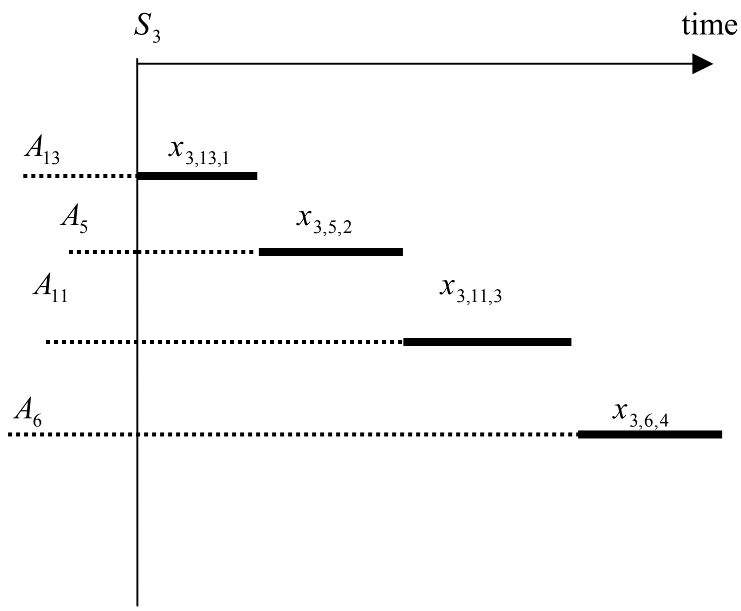

Figure 3. Ships served at berth 3.

In Figure 3., A_{13}, A_5 , A_{11} and A_6 denote the arrival times of ships 13, 5, 11 and ship 6, respectively. $x_{3,13,1}$ represents a decision variable, which corresponds to ship 13 being served at berth 3 as the 1st ship. $x_{3,13,1}$ apparently is set to 1 (once and only once in that position and otherwise it produces the value 0 in all other entries). Similarly, $x_{3,5,2}$ corresponds to ship 5 at berth 3 as the 2nd ship, $x_{3,11,3}$ to ship 11 at the same berth as the 3rd ship and $x_{3,6,4}$ to ship 6 as the 4th ship. S_3 denotes the start time of the availability of berth 3 in the present planning horizon. Dotted lines represent the waiting time for a given vessel, while thick line segments correspond to the handling times of particular vessels. This can be put into notations with $C_{3,13}$, $C_{3,5}$, $C_{3,11}$ and $C_{3,6}$ for vessels 13, 5, 11 and 6, respectively at berth 3.

The waiting time for vessel 13 will be $S_3 - A_{13}$, since it is the first vessel to be served at berth 3. Similarly, the waiting time for ship 5 will be $S_3 - A_5$, but it should include the handling time of its predecessor, ship 13, $C_{3,13}$, resulting in $S_3 - A_5 + C_{3,13}$. In the same manner, the total waiting time for ship 11 is $S_3 - A_{11} + C_{3,13} + C_{3,5}$ and that of ship 6 is $S_3 - A_{11} + C_{3,13} + C_{3,5} + C_{3,11}$.

Based on mathematical expression of waiting time of ship, the problem can be formulated as:

Minimise $$H = \sum_{j \in V} \sum_{k \in U} C_{Qj} x_{Qjk} \tag{3-1}$$

subject to $$\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V \tag{3-2}$$

$$\sum_{j \in V} x_{ijk} \leq 1 \qquad \forall i \in B, k \in U \tag{3-3}$$

$$\sum_{i(\neq Q) \in B} \sum_{k \in U} \left\{ S_i - A_j + \sum_{l \in V} \sum_{m \in P_k} C_{il} x_{ilm} \right\} x_{ijk} \leq L \tag{3-4}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \qquad (3\text{-}5)$$

where L is the stipulated waiting time and x_{ijk}'s are decision variables.

The objective function, given by (3-1), aggregates the handling time of each ship directed to the external terminal (C_{Qj}). This minimises the sum of the handling time associated with the ships which are allocated to the external terminal Q .

Constraints (3-2) give the assurance that all the ships, which have been called to the port should be served only once at any berth, including the external terminal, in any order of service. Constraints (3-3) ensure that each berth serves at most one ship at a time. Constraint (3-4) guarantees that ships cannot wait more than the stipulated waiting time L. The first term in the left hand side of constraint (3-4), $S_i - A_j$, gives the waiting time of a particular ship up to the planning horizon. It should be noted that as $S_i \geq A_j$ for all *i* and *j*, $S_i - A_j \geq 0$. The second term gives the accumulated handling time of predecessors of the vessel as a part of its waiting time.

It is considered that while there is no waiting time limit criterion with respect to the external terminal, the vessels directed to that terminal do not wait for more than *L* either. Practically, if there is no waiting time at all for the external terminal then there will be more vessels allocated to the external terminal due to that fact and hence it would apparently result in queuing up vessels for the external terminal. This is, however, unlikely to happen due to the objective function.

<u>3.3) Formulation of DEBAP:</u>

In practical situations, ships do not necessarily call at a port before a planning horizon. It may very well happen that some ships do call before and during a planning horizon while other ships may call while work is in progress. In this scenario, some ships may wait some time to be allocated to berths whereas other ships may be berthed on arrival without any waiting time, depending on the availability of berths.

Assumptions made to analyse DEBAP are:

- There are no constraints such as ships' draft and length
- All the ships can be berthed at any berth.
- Ships can arrive at any time to be served.
- The handling time of each ship depends on the berth in which it is served.
- Handling times of each ship at each berth are known.

As in the case of SEBAP, allocation of berths can be illustrated with the rectangular boxes represented by Figure 4. The difference here is that, since there could be vessels that arrive after the given planning horizon, some berths may fall vacant and can be idle for some time without a vessel at that particular berth. Therefore, there may be some time-gaps in between completion of handling a vessel and berthing a subsequent vessel.

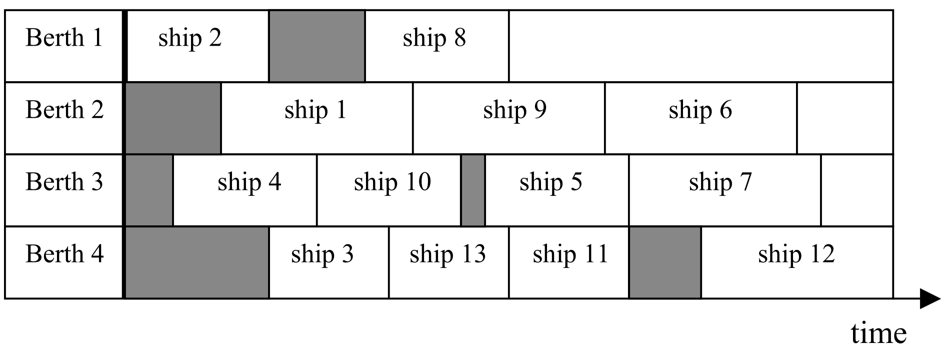

Figure 4. Allocation of berths.

Blocks in ash colour in Figure 4. represent the time gaps between completion and berthing of vessels (idle times).

The following notations, in addition to the notations used in SEBAP, are used in formulating DEBAP.

W_i : subset of ships with $S_i \leq A_j$

y_{ijk} : idle time of berth i between the departure of the k-1 th ship and the berthing of the k th ship when the ship j is served as the k th ship

For the simplicity in understanding, berth 3 in Figure 4 should be considered and the facts using a Gantt chart should be analysed. Figure 5 represents the vessels allocated to berth 3, including idle times between berthing and completion of vessels.

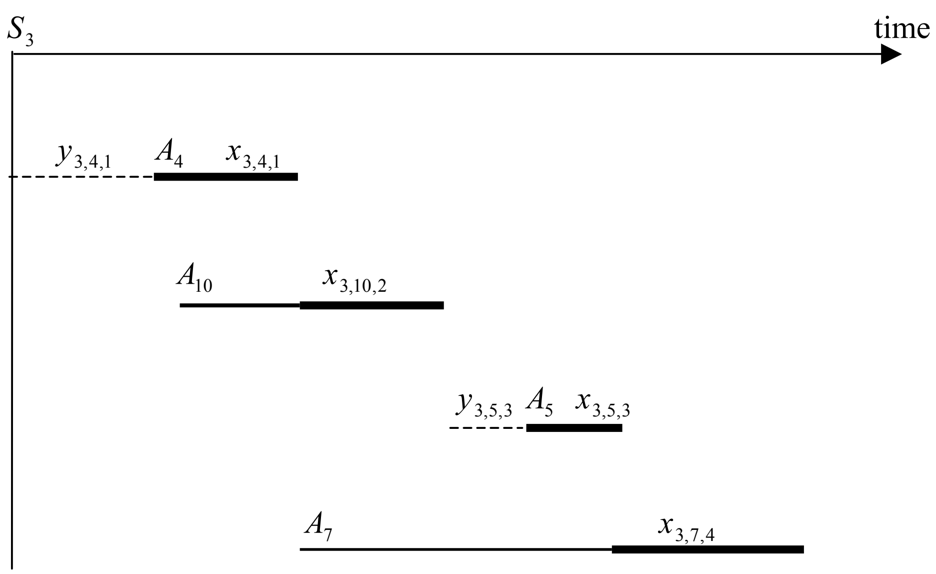

Figure 5. Ships served at berth 3.

At berth 3, berthing arrangement is done for ship 4 as the first one, ship 10 as the 2nd ship, ship 5 as the 3rd and finally ship 7 as the 4th. Figure 5 shows that berth 3 is idle before berthing ship 4 as the 1st ship to be served, which is represented by $y_{3,4,1}$. In the same manner, an idle time before berthing ship 5 is given by $y_{3,5,3}$. The handling times of these ships are denoted by $C_{3,4}$, $C_{3,10}$, $C_{3,5}$ and $C_{3,7}$. Since ship 4 is berthed on arrival, its waiting time is 0. Ship 10 has got a waiting time of $A_4 + C_{3,4} - A_{10}$ including the handling time of its predecessor, ship 4. As the berth has fallen idle after the completion of ship 10, ship 5 is berthed on arrival so that it does not have any waiting time either. Ship 7 has got a waiting time of $C_{3,10} + y_{3,5,3} + C_{3,5} - A_7$. It should be noted that the berth idle time between the completion time of ship 10 and the berthing time of ship 5, is included in the waiting time of ship 7. Also the handling times of both its predecessors, ships 10 and 5, are added up.

As stated above, ships may be handled at allocated berths after their arrivals, which is an additional constraint required for DEBAP. According to Imai et al. (2003), it could be formulated as follows.

Minimize $$H = \sum_{j \in V} \sum_{k \in U} C_{Qj} x_{Qjk} \tag{3-6}$$

subject to $$\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V \tag{3-7}$$

$$\sum_{j \in V} x_{ijk} \le 1 \qquad \forall i \in B, k \in U \tag{3-8}$$

$$\sum_{i(\neq Q) \in B} \sum_{k \in U} \left\{ S_i - A_j + \sum_{l \in V} \sum_{m \in P_k} C_{il} x_{ilm} \right\} x_{ijk} + \sum_{i(\neq Q) \in B_i} \sum_{k \in U} \left\{ y_{ijk} + \sum_{l \in V} \sum_{m \in P_k} y_{ilm} \right\} x_{ijk} \le L \qquad \forall j \in V \tag{3-9}$$

$$\sum_{l \in V} \sum_{m \in P_k} (C_{il} x_{ilm} + y_{ilm}) + y_{ijk} - (A_j - S_i) x_{ijk} \ge 0 \quad \forall i \in B, j \in W_i, k \in U \tag{3-10}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \qquad (3\text{-}11)$$

$$y_{ijk} \geq 0 \qquad \forall i \in B, j \in V, k \in U \qquad (3\text{-}12)$$

where x_{ijk}'s and y_{ijk}'s are decision variables.

Constraint (3-10) can be re-written by taking the last term on the left hand side of the inequality to the right hand side, as:

$$\sum_{l \in V} \sum_{m \in P_k} (C_{il} x_{ilm} + y_{ilm}) + y_{ijk} \geq (A_j - S_i) x_{ijk}$$

This represents the values for the ship *j* at berth *i*, when it is served as the *k* th ship in the service order. As term $C_{il} x_{ilm} + y_{ilm}$ gives the handling time and the idle time (if any) of predecessors of a ship, the sum of the term over the predecessors gives the time gap between S_i and the last of its predecessor leaving the port. Therefore, if x_{ijk} = 1, then the time gap between S_i and the start time of servicing vessel *j* should not be less than $A_j - S_i$. It means, the constraints ensure that no ship is served before its arrival.

Consider constraint (3-9):

$$\sum_{i(\neq Q) \in B} \sum_{k \in U} \left\{ S_i - A_j + \sum_{l \in V} \sum_{m \in P_k} C_{il} x_{ilm} \right\} x_{ijk} + \sum_{i(\neq Q) \in B_i} \sum_{k \in U} \left\{ y_{ijk} + \sum_{l \in V} \sum_{m \in P_k} y_{ilm} \right\} x_{ijk} \leq L.$$

The first term on the left hand side is the same as in the case of SEBAP, except for the fact that, in this case, A_j can be greater than S_i. With the explanation of constraint (3-10), positivism of the first term of the constraint under consideration can be seen and further that term accumulates all the handling times and gives the time strip of the particular vessel between the arrival and S_i. The second term on the left hand side accumulates all the idle times, if any, at that particular berth. Altogether, the sum of all the cited terms has to be less than the stipulated time L.

CHAPTER FOUR

SOLUTION ALGORITHM

In this chapter a solution algorithm is discussed to solve the problems, under consideration; the SEBAP and DEBAP.

4.1) Introduction:

In any given port, according to the scheduling of the berthing arrangements, ships would experience different waiting times depending on many factors. Some of them have already been identified in the earlier chapters while a few of them can be put forward as followings.

1. Arrival time of the vessel.
2. Time of an assigned berth fall vacant.
3. Handling and completion times of the preceding vessels.
4. Handling rates (productivity) of the ports.
5. Congestion in the port.
6. Other possible natural and artificial delays.

Since the scope of this study is more related to the facts (1), (2) and (3), only these things have been taken into account in the solution procedures.

For the above three reasons, there could be different waiting times associated with different ships in SEBAP as well as in DEBAP. Therefore first the vessels which have waiting times and those which do not have, to be distinguished depending on the case. Then, those with long waiting times have to be selected among ships with waiting time, in order to see whether they violate the stipulated time criterion (L)

defined by the terminal operator. For this task, first a pool of such vessels is formed so that the attention could be focused on only that pool of vessels.

The aim here is that the allocation of vessels to the external terminal has to be a minimum to reduce the costs associated with vessels to be served at the external terminal. This can be done in three ways.

1. Allocation of least possible number of vessels irrespective of the handling times of those at the external terminal.
2. Allocation of least number of vessels with longer handling times at the external terminal.
3. Allocation of vessels to the external terminal, taking into account the minimum handling times at the external terminal.

According to the discussion in the introduction, one may recall that the costs associated with such an assignment, for JCT, would be too heavy if there are a large number of containers to be handled at the external terminal. Therefore, handling times at the external terminal have to be taken into account in the light of costs associated with it and hence option (1) would not be the ideal criterion. Similarly, if the option (2) were considered, even though the number of vessels may be minimum, still the handling times would be very long, resulting in considerable costs imposed to the JCT. Hence, option (2) may neither be suitable. When one considers the minimum handling time of vessels, which are directed from the JCT, at the external terminal, then only the JCT would bear the minimal cost with regards to the above-mentioned operation. In that regard, option (3) may be an ideal criterion to be adopted.

Consequently, a methodology should be framed to direct the vessel/vessels with minimum handling time/times at the external terminal, selecting from the pool of vessels those violating the time limit criterion stipulated by the JCT.

4.2) Obtaining an initial berth allocation:

In the entire solution algorithm, a preliminary berth allocation should first be obtained, where some vessels may violate the stipulated waiting time L. This allocation is found by solving the following Assignment Problem (Bronson, Naadimuthu, 1997; Daellenbach, George, McNickle, 1983; Wayne, 1994) formulation.

$$\text{Minimise} \quad H = \sum_{j \in V} \sum_{k \in U} C^{*}_{ij} x_{ijk} \tag{4-1}$$

$$\text{Subject to} \quad \sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V \tag{4-2}$$

$$\sum_{j \in V} x_{ijk} \le 1 \qquad \forall i \in B, k \in U \tag{4-3}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \tag{4-4}$$

where $C^{*}_{ijk} = (T-k)C_{ij} + S_i - A_j$

This formulation minimises the total waiting time of vessels in SEBAP. Consequently, it tries at the same time to minimise the number of vessels being directed to the external terminal. In the case of DEBAP, the solution to the formulation is modified so that the services of vessels are postponed to the times of arrival. While various exact solution methods have been proposed by a lot of researchers, Mack's method (Bunday, 1984) was applied in this study to find the optimal solution to the Assignment Problem.

4.3) Obtaining feasible berth allocation with violating vessels assigned to the external terminal:

Based on a solution by the formulation (4-1)-(4-4), a pool is initialised with all the vessels violating the stipulated waiting time L. then the solution is modified by the following procedure shown in Figure 6, so that all the vessels to be served at the own terminal do not wait more than L.

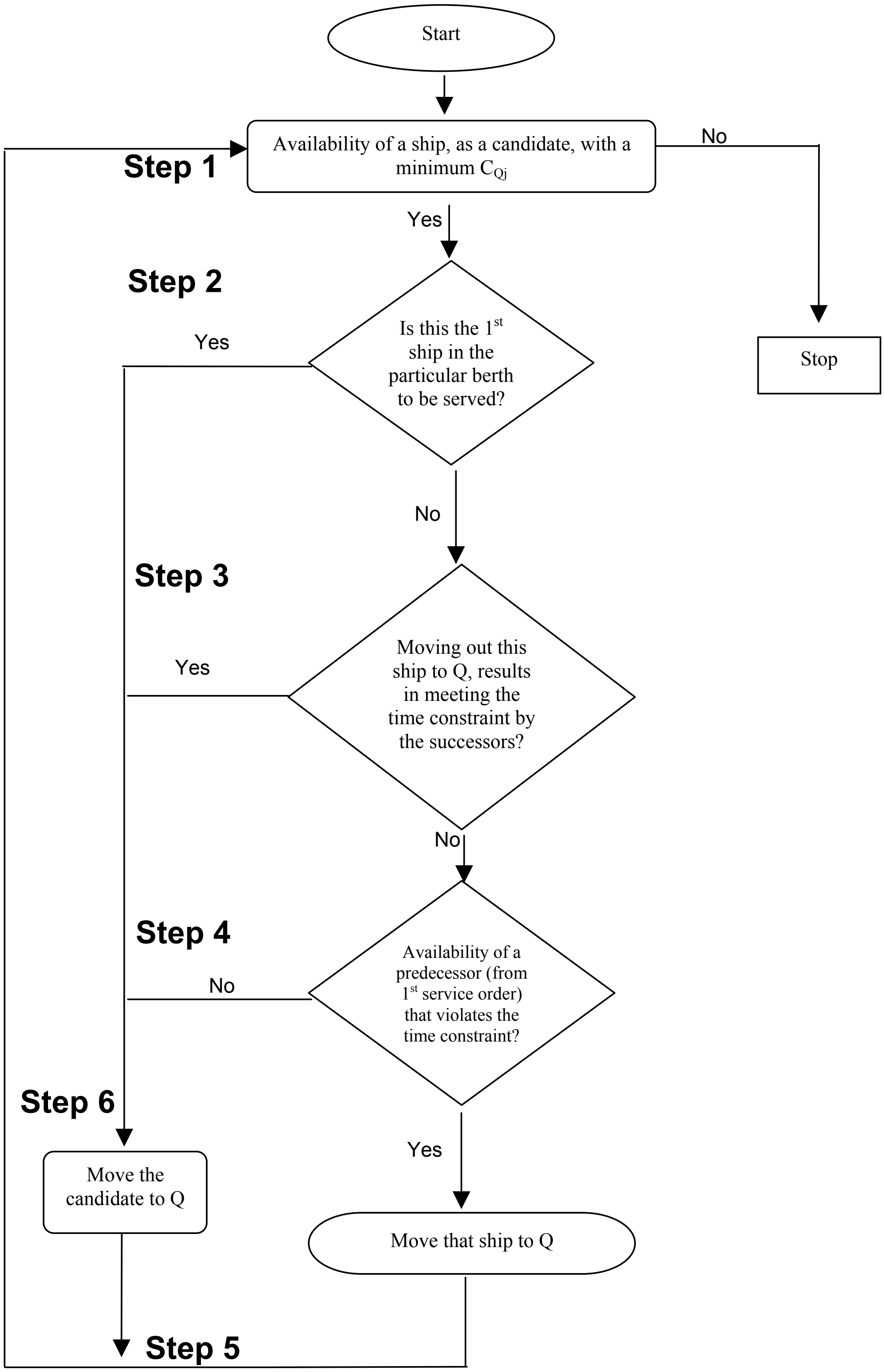

Figure 6. Solution algorithm.

4.4) Illustrated explanation of the algorithm:

a) SEBAP:

In this case, all the vessels have arrived before a given time horizon. For the convenience of understanding, a single berth with a random assignment of vessels should be considered. (See Figure 7).

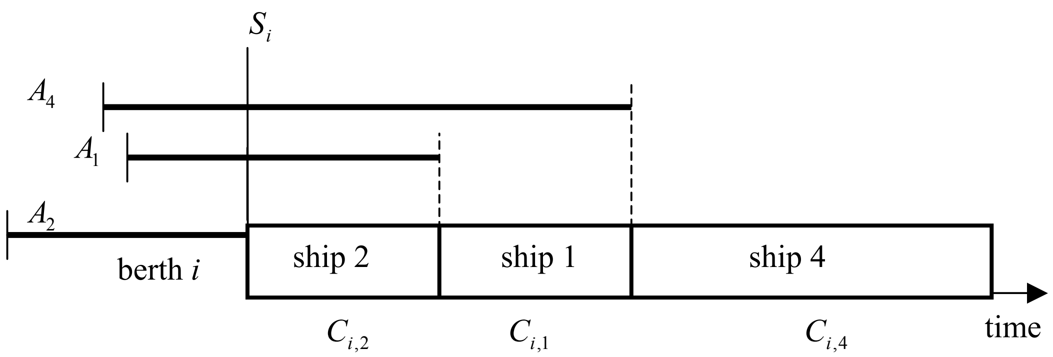

Figure 7. Berth i.

A_1, A_2, A_4 represent the arrival times and $C_{i,1}$, $C_{i,2}$, $C_{i,4}$ represent the handling times of ships 1, 2 and 4 and their service orders (k) being 1, 2 and 3 respectively at berth i. Planning horizon is S_i .

Waiting time for;

Ship 2. $= S_i - A_2$

Ship 1. $= (S_i - A_1) + C_{i,2}$

Ship 4. $= (S_i - A_4) + C_{i,2} + C_{i,1}$

It should be noted that the handling times of vessels accumulate for the waiting times of their successors.

Suppose that waiting times for ships 1 and 4 are greater than L. That is,

$(S_i - A_1) + C_{i,2} > L$ and

$(S_i - A_4) + C_{i,2} + C_{i,1} > L$.

Then the pool would consist of ships 1 and 4. Further supposing that, according to Figure 7 above, the handling times of those vessels at the external terminal are $C_{Q,1}$ and $C_{Q,4}$ respectively where $C_{Q,1} < C_{Q,4}$.

In Step 1 of the algorithm as shown in Figure 6, a vessel is selected as a candidate from the pool that has the minimum handling time at the external terminal. If no candidate is found, then the procedure is terminated. If a candidate is available, then proceed to Step 2. In the example, candidate is ship 1, since $C_{Q,1} < C_{Q,4}$.

Then in Step 2, it should be checked whether ship 1 is the first ship to be served at berth *i*. If this is the first ship, then it should be directed to the external terminal as done in Step 6. In the example, that ship does not satisfy the condition, thus Step 3 should be followed.

In Step 3, it is examined if the removal of the ship makes its successor(s) meet *L* criterion. The removal of ship 1 results in the following rearrangement of the service of its successor(s) as portrayed in Figure 8.

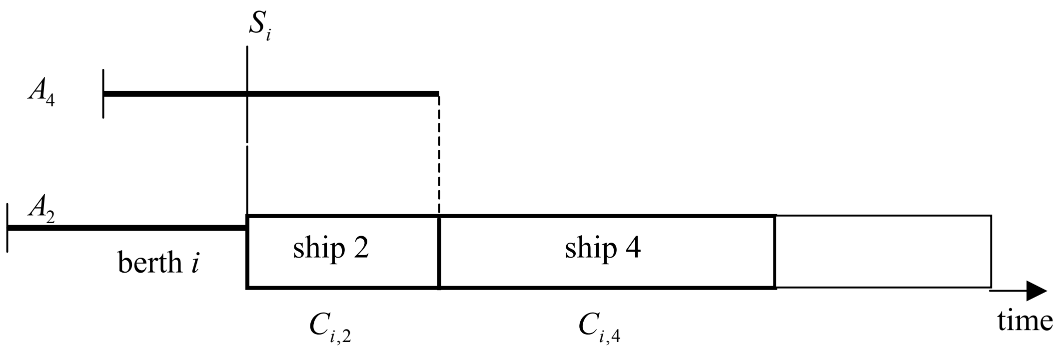

Figure 8. Modified arrangement of serving ships at berth *i*.

This results in recalculating the waiting time of the succeeding ships in the pool. By the recalculation the waiting time of ship 4 becomes;

(Ship 4.)$_{\text{Modified}}$ = $S_i - A_4 + C_{i,2}$

The resulting service orders for ships 2 and 4 are 1 and 2 respectively.

If $S_i - A_4 + C_{i,2} \leq L$

then, ship 1 is removed from the pool and Step 6 is followed.

If $S_i - A_4 + C_{i,2} > L$

then, proceeding to Step 4, its predecessor(s) should be looked into. The check is performed from the 1st ship at the berth.

If the predecessor violates the time limit criterion *L,* then ship 2 is to be directed to the external terminal as done in Step 5. But in the example, it does not violate *L* and therefore ship 1 has to be directed to the external terminal as done in Step 6.

After completing one phase of directing a ship to the external terminal, the waiting times of the succeeding ships of the removed one are recalculated. If the recalculation results in the satisfaction of *L* criterion for some ships, they are removed from the pool.

What is left then is the same situation represented by Figure 8. Next step is to go to the pool with the remaining vessels in it to find another candidate, if there is any. If no vessel is violating the time limit criterion, then the final berth assignment of berth *i* will take the form of Figure 8 above.

In the example, it is supposed that $S_i - A_4 + C_{i,2} > L$; therefore the vessel with the minimum handling time at the external terminal is ship 4 since it is the only vessel left in the pool. As it is not the first ship at the berth *i* to be served and has no successors, its predecessor(s) should be checked. As no preceding vessel can be found with violation vessel 4 is to be directed to the external terminal. The resulting ship assignment at berth *i*, is shown in Figure 9.

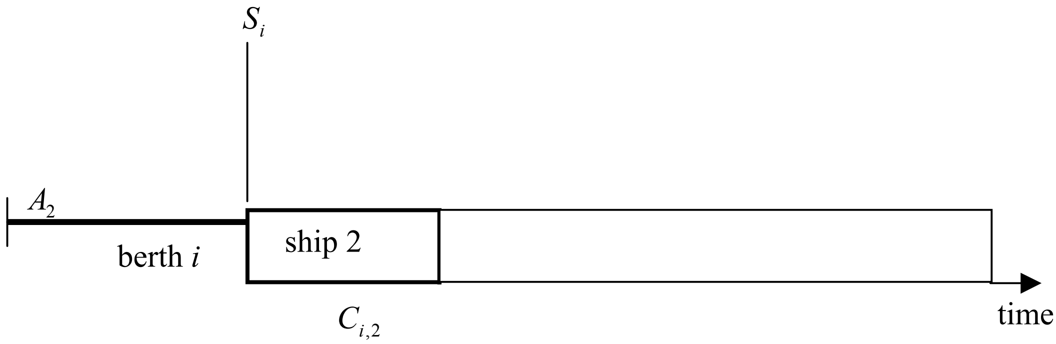

Figure 9. Final assignment at berth *i*.

<u>b) DEBAP</u>:

In DEBAP, vessels may arrive at any time, not necessarily calling before a given planning horizon. Berths may fall vacant at different instances depending on the handling times of preceding vessels, and further there may be some time gaps (idle times) in between two consecutive services of vessels. This scenario may differ from berth to berth having been influenced by the different handling times of vessels and number of calling vessels. Therefore, some vessels, which may arrive at the port while work is in progress, may not experience waiting times at all, whereas some may have relatively shorter waiting times, without the stipulated time violation. Even though a vessel arrives at a sufficiently earlier time, before a berth fall vacant, it may not necessarily be berthed without suffering from a waiting time.

Consider ship services as illustrated in Figure 10.

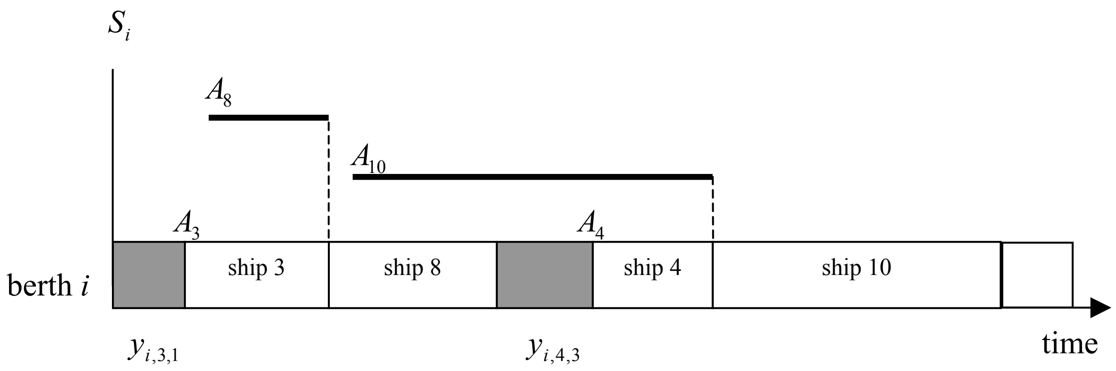

Figure 10. Berth *i*.

Apart from the notations which have been used in the SEBAP, $y_{i,3,1}$ and $y_{i,4,3}$ represent the idle times of berth *i*. $y_{i,3,1}$ defines an idle berth between the start of the present horizon and ship 3, while $y_{i,4,3}$ is one between ships 8 and 4. Letting the handling times of vessels 3, 8, 4, and 10 at berth *i* be $C_{i,3}$, $C_{i,8}$, $C_{i,4}$ and $C_{i,10}$ respectively.

Waiting times for those ships are;

Ship 3 = 0 (since it is berthed on arrival).

Ship 8 = $A_3 + C_{i,3} - A_8$.

Ship 4 = 0 (since it is berthed on arrival).

Ship 10 = $A_4 + C_{i,4} - A_{10}$.

Now suppose that ships 3 and 4 do not violate *L* (obviously they have been berthed on arrival) while ships 8 and 10 violate *L* and that their respective handling times at the external terminal are $C_{Q,8}$ and $C_{Q,10}$ where $C_{Q,8} < C_{Q,10}$. Then the pool evidently consists of ships 8 and 10. As the first candidate, ship 8 is selected since it is the ship, in the pool, with the minimum handling time at the external terminal.

If this ship were the first one to be serviced at berth *i*, it would be directed to the external terminal, according to the solution algorithm. Since it is not the case, it has to be checked whether directing of this candidate to the external terminal results in meeting the time limit criterion by its successors who violates L. If this is removed to the external terminal, the modified arrangement of berth i is the one portrayed by Figure 11.

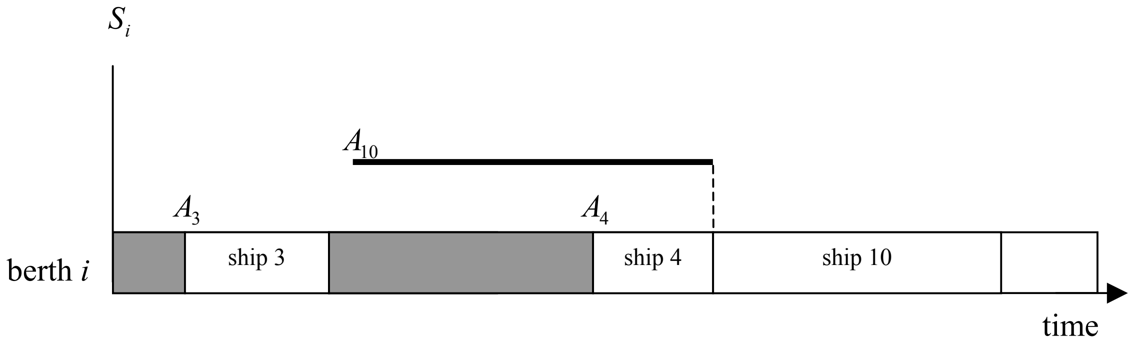

Figure 11. Modified arrangement of serving ships at berth i.

Even though this removal results in a long idle time after the completion of ship 3, ship 4 is not put towards the ship 3 because of A_4. For the simplicity, the algorithm does not move ship 10 from the present position in time to the idle time before ship 4. Therefore, the removal of ship 8 does not result in meeting the time constraint by its successors, ship 10. This concludes ship 8 does not seem to be moved to the external terminal so far. However, as ship 3, which is the only predecessor of ship 8, is not a violating ship, it is removed. Subsequently berth i would show the same structure as shown in Figure 11.

Next the same process is repeated for ship 10, which is another violating vessel. As it is the last one in the pool, it is simply removed from the pool and directed to the external terminal.

CHAPTER FIVE

COMPUTATIONAL EXPERIMENTS

In this chapter, mainly the computational experiments with regard to the main objective of minimizing the vessels directed to the external terminal *Q* are discussed. The main inputs to computer programmes implemented for the experiments are also looked into.

5.1) Overview of computer programmes:

The programme consists of four sub programmes, which are:

a. BAP-AP.
b. CHGSOL.
c. SBAP.
d. DBAP.

These sub-programmes coded by *Microsoft Visual Basic (VB)* programming language run on *Microsoft Excel*. Microsoft Excel was used to do all the necessary computations in order to format the input data so as to be compatible with the *VB* programmes. For the experiments, arrival times and handling times of 61 vessels, which were observed at JCT of the port of Colombo, Sri Lanka, were taken into consideration.

a) BAP-AP:

This programme calculates the preliminary berth allocation to ships involved, which is in section 4.2. The solution of the optimum berth allocation of vessels depends on the different handling times at each berth for each vessel. It requires, as the input, the number of vessels per schedule and the number of berths in which vessels to be assigned. For example, if there are 10 ships and 3 berths, then the input has to take the form of a table where the different handling times of those vessels are calculated according to the berth assigned to them. Further, 10 different service orders also have to be calculated for each berth, since it may very well happen that, in an extreme case, all the 10 vessels may be assigned to the same berth leaving all the other 2 berths entirely free of any operation. It should be noted that a set of berths must include the external terminal; however the solution should have as few vessels assigned to the external terminal as possible. For this, regarding the external terminal as a dummy berth in the computation, very long handling times of vessels are associated with the dummy berth.

BAP-AP considers the optimal allocation of vessels taking into account the least possible handling times of vessels at different berths. BAP-AP produces an output so that each vessel to only one berth and a berth is engaged only by one vessel at a time.

b) CHGSOL:

An output of BAP-AP is given by decision variables x_{ijk}. This format of the solution is not tractable for the subsequent process. The programme CHGSOL converts the solution of BAP-AP to a solution in the form of the Gantt chart as shown in chapter 3. More exact form of the solution is illustrated in Figure 12, where figures in the matrix are vessel numbers.

Service order	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Berth 1	6	5	7	0	0	0	0	0	0	0
Berth 2	10	1	2	8	0	0	0	0	0	0
Berth 3	3	9	4	0	0	0	0	0	0	0

Figure 12.Berth assignment.

Figure 12. describes that berth 1 is assigned with vessels 6, 5 and 7 as the 1st, 2nd and 3rd vessels at that berth and berth 2 with vessels 10, 1, 2 and 8 as the 1st, 2nd, 3rd and 4th and berth 3 is assigned with vessels 3, 9 and 4 as the 1st, 2nd and 3rd respectively. It should be noted that as no vessels are obviously served after the last vessel to be served at each berth, service orders after the last one are filled with nullity. Also, each berth has been considered up to the 10th service order taking into account the extreme case mentioned above. A typical representation of this berth assignment by a Gantt chart is shown in Figure 13.

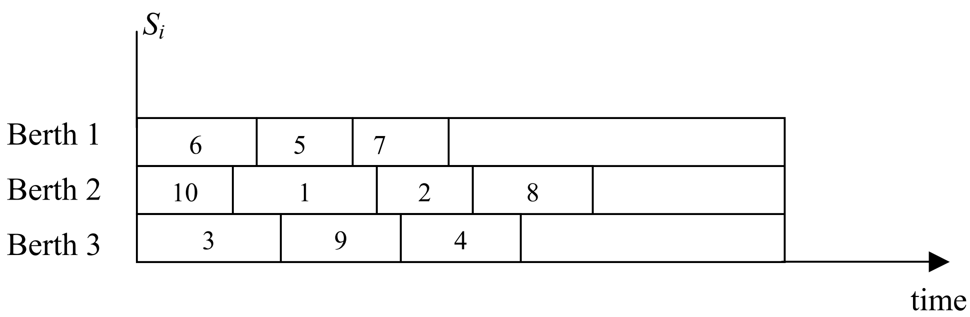

Figure 13. Berth assignment by the Gantt chart.

S_i for all three berths may not be the same depending on the time of a particular berth fall vacant immediately before a given planning horizon. Even though not shown in the chart, there could be some idle times in between two consecutive vessels in DEBAP.

c) SBAP/DBAP:

Both programmes, SBAP for SEBAP and DBAP for DEBAP, require, as inputs, the above data set in Figure 12. In addition to that, it needs another input data with arrival times of vessels and handling times at each berth including the external berth. The latter takes the following form as shown in Figure 14 and 15 depending on the case, both of which have 10 vessels and 3 berths.

S_i	18									
A_j	8	11	6	13	7	12	5	9	15	16
$C_{1,j}$	5	18	8	13	5	5	12	2	6	5
$C_{2,j}$	6	19	9	14	6	6	13	3	7	5
$C_{3,j}$	7	20	10	15	7	7	14	4	8	6
$C_{Q,j}$	7	20	10	15	7	7	14	4	8	6

Figure 14. Input data set for SBAP.

S_i	13	19	16							
A_j	12	15	18	25	27	31	31	33	42	42
$C_{1,j}$	5	18	8	13	5	5	12	2	6	5
$C_{2,j}$	6	19	9	14	6	6	13	3	7	5
$C_{3,j}$	7	20	10	15	7	7	14	4	8	6
$C_{Q,j}$	7	20	10	15	7	7	14	4	8	6

Figure 15. Input data set for DBAP.

It should be noted that in Figure 14. S_i is greater than all A_j's because of the SEBAP case, where all the vessels have arrived before a given planning horizon. The planning horizon in this case spreads over 1800 hrs.

In Figure 15 some arrival times of ships (A_j) are greater than the times of berths falling vacant (S_i). This reflects that those vessels are arriving while work is in progress. As there are 3 berths, namely 1, 2 and 3, they fall vacant at different times of 1300, 1900 and 1600 hours respectively.

Arrival times of ships have been identified as follows:

1. Taking the first day of the data set as the starting date, any vessel which arrived on that day, has the arrival time less than or equal to 24.
   - E.g. first three vessels in Figure 15 have arrived at 1200, 1500 and 1800 hours correspondingly.
2. When the vessels scheduled for the second day are considered, a value of 2400 hours is added and for third day 4800 hours and so on.
   - E.g. fourth vessel in Figure 15 has arrived at 0100 hours on the second day and therefore it has the value 0100+2400=2500 hours or 25 for the easy calculation. The last vessel in Figure 15 has arrived at 1800 hours on the second day and therefore has the value 1800+2400=4200 hours or simply 42 and so on.

5.2 Input data for the experiments:

Table 1 shows the data set, which was used to evaluate the whole analysis.

Table 1. Input data set.

Vessel number	**Time of arrival**	**Handling times at each berth**				
		Berth 1	**Berth 2**	**Berth 3**	**Berth 4**	**Ex: Berth Q**
1	12	5	6	7	8	7
2	15	18	19	20	21	20
3	18	8	9	10	11	10
4	25	13	14	15	16	15
5	27	5	6	7	8	7
6	31	5	6	7	8	7
7	31	12	13	14	15	14
8	33	2	3	4	5	4
9	42	6	7	8	9	8
10	42	5	5	6	7	6
11	43	11	11	12	13	12

12	41	6	6	7	8	7
13	57	8	8	9	10	9
14	58	7	7	8	9	8
15	61	9	9	10	11	10
16	63	5	5	4	3	4
17	65	5	5	4	3	4
18	70	5	5	4	3	4
19	76	12	12	11	10	11
20	89	9	9	8	7	8
21	79	15	15	14	13	14
22	85	14	14	13	12	13
23	85	8	8	7	6	7
24	94	13	13	12	11	12
25	96	8	8	7	6	7
26	92	10	10	9	8	9
27	108	6	6	5	4	5
28	104	6	6	7	8	7
29	108	2	2	3	4	3
30	108	5	5	6	7	6
31	109	7	7	8	9	8
32	112	7	7	8	9	8
33	128	19	19	20	21	20
34	129	5	5	6	7	6
35	138	2	2	3	4	3
36	141	1	1	2	3	2
37	150	4	4	5	6	5
38	148	10	10	11	12	11
39	150	3	3	4	5	4
40	150	4	4	5	6	5
41	150	9	9	10	11	10
42	159	11	11	12	13	12
43	159	5	5	6	7	6
44	162	8	8	9	10	9
45	164	6	6	7	8	7
46	162	9	9	10	11	10
47	165	7	7	8	9	8
48	176	4	4	5	6	5
49	175	8	8	9	10	9
50	185	6	6	7	8	7

51	202	7	7	6	7	7
52	211	2	2	1	2	2
53	223	17	17	16	17	17
54	224	5	5	4	5	5
55	234	8	8	7	8	8
56	226	7	7	6	7	7
57	224	6	6	5	4	5
58	239	6	6	5	4	5
59	244	4	4	3	2	3
60	238	8	8	7	6	7
61	239	11	11	10	9	10

In Table 1, columns 2 and 3, representing 'Time of arrival' and handling times at 'Berth 1' of vessels, are comprised of actual values. (In fact some of them were rounded-off to the nearest integer value to avoid tedious computations and for easy interpretations but still the real values are going to work without any problem). Practically there is no vessel, which is served at many different berths with identical parameters such as number of containers and time of arrival. In other words, when the real handling time of a vessel is known, imaginary parameters have to be applied to determine the handling time of the same ship at other berths with the same load of containers. In some practical problems the distance to the container yard, where the relevant containers are stored after unloading from or before loading on the vessel, determines the time taken to complete the handling of the vessel. This distance, certainly, would be related to the cycle times of the container transportation between the yard and ship, giving way to different operational times at dissimilar berths.

Therefore, handling times at other berths are to be calculated taking into account the effect of the additional distances experienced. In reality, there could be a difference of one or two hours according to practical experiences gained through day-today terminal operations. With regard to the handling times at the external terminal, an average handling time has been calculated for each vessel in view of the different handling times at four other berths for the same vessel. Handling times at the external terminal *Q* are given in the last column of Table 1.

As the SEBAP is a special case of DEBAP, it is convenient to concentrate on DEBAP. As a result, discussion henceforth is pursued on DEBAP.

Under normal circumstances, in any port, the berth assignment is done on a periodic basis. Any particular port, depending on many factors, determines that period of time. This period may have effects due to the traffic patterns arising from

- Eastbound or westbound – far East- Europe services etc,
- Type of vessels calling – mother/main liner vessels or feeder vessels,
- Frequency of cargo consolidation,
- Type of port-hub port or feeder port,
- Port performances etc.

Due to these various facts consequential to many explicit and implicit reasons, many ports opt to adopt scheduling methods in which the scheduling period for berthing would be less than one week. Further, in those ports that are operating as busy hubs wherein tight schedules are maintained due to the congested situations, there may be instances that they plump for 3 days or 4 days, even sometime 2 days berth scheduling. The whole idea behind this type of berth planning may be, even according to the realistic experiences, to accommodate as many vessels as they could without hampering the sailing schedules of vessels. This may very well depend on the availability of number of berths for operation. Experience shows that, in that type of berthing segmentations, only about 8 to 20 vessels were involved in berthing schedule preparation.

When the actual figures obtained, from a busy hub port, regarding arrival patterns of vessels to be served in that particular port, a smooth pattern of vessel calls on each day, could not be found. There was a big difference in number of calling vessels during a specified sequence of periods of time. When the figures of a period of one week were compared with the following week, there was not any relation between these two. Moreover, even consecutive days do not demonstrate a similar pattern regarding the number of calling vessels.

Consequently, there could be slack periods and taut periods in stints of allocation of berths when the planning is done based on number of days. Therefore, in order to have an even scrutiny about the analysis that is going to be made, splits of the entire scheduling period were chosen based on number of calling vessels. The foremost idea behind this is to see which split of vessels could be more suitable to the maximum possible level of berth scheduling.

These splits were designed by taking into account the data set gathered and the total number of vessels involved. The data set contains 61 incoming vessels. Within the 61 vessels, during the period of the data set collected an identical vessel may be included as two or more different port calls with different identification numbers of vessels.

In view of the fact that there are 4 berths in the own terminal, assuming two vessels to be served at each berth per day, the largest number of splits with 8 vessel calls per split is obtained. This average number was found by the fact that, under normal circumstances a vessel may wish to have a time of about 12 hours in port, at most, per port call due to the sailing schedule commitments. Table 2 gives the splits and the corresponding number of vessels in each split.

Table 2. Scheme of split.

	Number of splits	**Number of vessels in a split**	**Number of vessels in the last split**	**Total**
1	8	8	5	61
2	7	9	7	61
3	6	10	11	61
4	5	12	13	61
5	4	15	16	61
6	2	30	31	61
7	1	61	61	61

Because of the total number of vessels and the number of splits, all the splits in a particular scheme do not necessarily have the identical number of vessels. Thus the number of vessels in the last split may have a smaller or larger number of vessels, while this does not deviate largely affecting the analysis.

5.3) Experiments:

All the calculations were performed setting the time limit L to be 4 hours. This limit of 4 hours would purely be calculated from the time of arrival of a vessel until the expected time of berthing. Table 3 provides a summary of the analysis under different schemes of splits.

Table 3. Summary of the results.

	Split	Own terminal			External terminal	
		Number of vessels	**Total waiting time (hours)**	**Average waiting time (hours)**	**Number of vessels**	**Total handling time (hours)**
1	8	42	31	0.74	19	218
2	9	41	20	0.49	20	225
3	10	42	26	0.62	19	210
4	12	41	33	0.80	20	223
5	15	41	27	0.66	20	236
6	30	35	11	0.31	26	255
7	61	42	26	0.62	19	205

It should be noted that the waiting times at the external terminal are not included in the table, since it is assumed that the vessels are to be served, at the external terminal, immediately and also it is not a main concern of this study.

When calculating the cases of various splits, completion time of the last vessel served at a particular berth was taken as the berth available time, S_i, for the next planning split. This process was repeated till the last split.

The main objective of this study is to minimise the costs associated with the operation of directing vessels to the external terminal. According to the discussion in chapter 1, those costs are directly related to the number of containers handled at the external terminal, in which containers have to be hauled to and from the external terminal from and to JCT. This in turn increases the dependency on the time taken to handle those containers at the terminal, while this can be termed as the total handling time at the external terminal. Therefore the explicit objective is to minimise the total handling time at the external terminal. Various values of the total handling time are shown with different scheme of splits in Figure 16, which was prepared from Table 3.

There is a trend that the handling time, at the external terminal, is increasing with more ships in a split of the planning horizon; however, interestingly the minimum handling time is found at the split with most number of ships.

While the objective of the problem is the minimisation of the total service time of vessels at the external terminal, it is interesting to look into the number of vessels directed to the external terminal. From this point of view, splits 8, 10 and 61 have the minimum number of vessels, as shown in Figure 17 that was prepared based on Table 3.

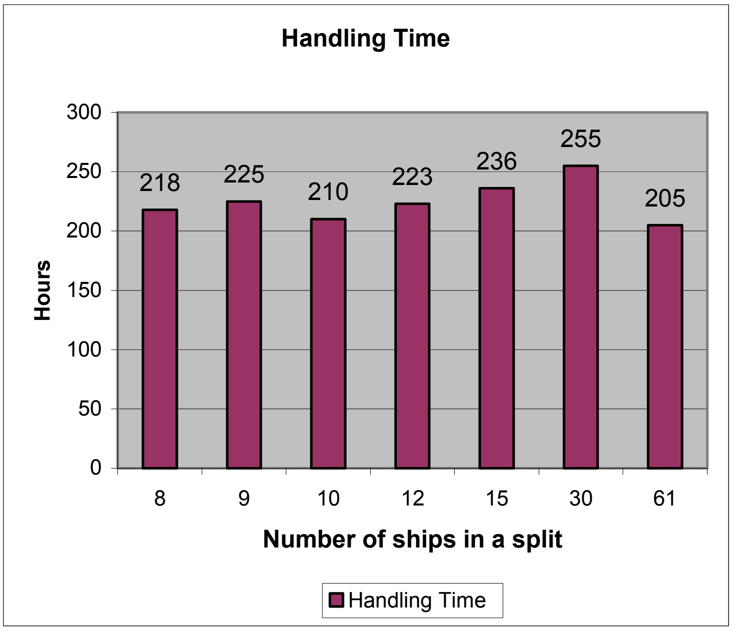

Figure 16. Handling times at the external terminal at each split.

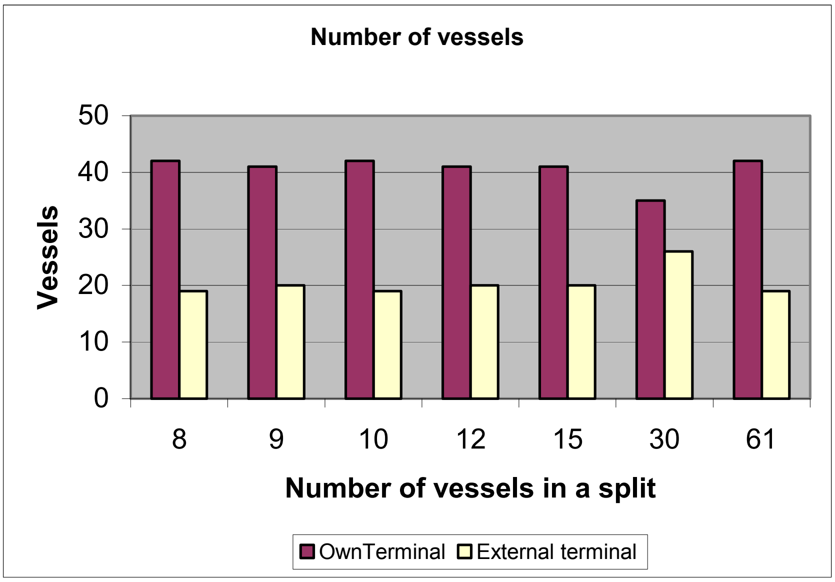

Figure 17. Number of vessels served at the own and external terminals.

Considering altogether with the handling time at the external terminal and the associated number of vessels, split 61 offers the most preferable solution. This planning scheme may require a very long planning horizon, about two weeks. It does not seem attractive from a practical viewpoint for the following reasoning: This scheme would correspond to the solution stretching over approximately 8 days, taking into account, generally, 8 vessels with 4 berths during an operational period of 24 hours per day. Such a long scheduling may necessitate a lot of changes in the final scheduling of berth allocation with some changes in ship arrival in the course of planning horizon, according to the 24 hour notices obtained from the shipping lines vis-à-vis the sailing schedules of vessels. This has been the common practice in almost all the commercial ports, paving way to a higher flexibility in berth planning. Therefore, this planning split of 61 vessels, in which many changes may likely take place, might perhaps not be that attractive in determining an operation that is aimed at minimizing all the costs associated with it. Such an ambiguous decision might result in huge costs if a situation would arise wherein many vessels might be directed to the external terminal due to these last minute changes in sailing schedules of vessels. This would be totally out of the control of any terminal and hence it would be better to avoid such a planning scenario.

Taking into consideration all these particulars, the best option could be the split of 10 vessels, because this split results in the second minimum handling time. Results obtained for this split are shown in Table 4.

Table 4. Details of split of 10 vessels.

Split	Own terminal			External terminal	
	Number of vessels	Total waiting time(hrs)	Average waiting time (hrs)	Number of vessels	Handling time (hrs)
10	42	26	0.62	19	210

In fact the maximum waiting time observed was 4 hours according to the statistics in JCT.

A larger amount of *L* would secure more vessels in the own terminal without directing them to the external terminal. Another aspect to be looked into is that the waiting times for the berths in the external terminal. This was exclusively avoided due to the fact that the main objective was in a totally opposite bearing. However it is noteworthy to reckon that once the ships are queued up to the berths in the external terminal, another waiting time factor would come into play. Therefore, the own terminal JCT would be in a position to fine-tune the waiting time criterion reasonably glowing in favour of the JCT, having further reduced the number of vessels directed to the external terminal SAGT, resulting in as less handling time in that terminal as possible.

Table 5. Waiting times and handling times.

Split	Own terminal	External terminal
	Average waiting time in minutes	Handling time in hours
8	44	218
9	29	225
10	37	210
12	48	223
15	39	236
30	18	255
61	37	205

It should be noted that in Table 5, among the three lowest average waiting times, i.e. 18, 29 and 37 minutes, the choice of the split of 10 vessels is still on a par with the earlier discussion and further in the cases of the lowest two of 18 and 29 minutes, other conditions were not contented as well as the main objective. In addition to that, most importantly, one of the lowest handling times too falls in the split of 10 vessels in which situation a vessel experiences an average waiting time of 37 minutes. Hence, an average waiting time, even though the average values do not always paint a true picture of a situation, of about 37 minutes, which cascades very well behind an hour, is not at all an unacceptable average waiting time for any vessel.

Following the selection of the number of splits, which proved to be the best among the available options, it is looked into how the solution is affected by various L's.

Those values are:

L = 3, 4, 5, 6 and 7.

It is intended to analyse the results of number of vessels handled at the own terminal and at the external terminal, waiting times and average waiting times at the own terminal and handling times at the external terminal with these various values of L.

Figure 18 illustrates the relationship between the total handling time at the external terminal and L, showing that when L is increasing, handling times at the external terminal is decreasing. However, a decreasing value with L from 3 to 4 is not as much as the one with L from 4 to 5. Notably there is no gap in the handling time between L of 5 and 6. But there is a significant difference in hours when L is increased from 6 to 7.

Figure 19 illustrates the number of vessels handled at each terminal, while Figure 20 shows the waiting time and the average waiting time per vessel.

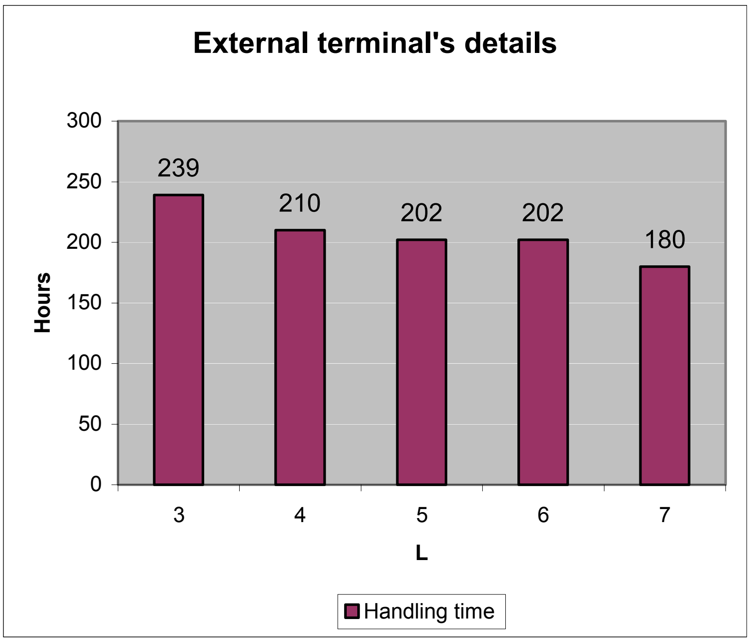

Figure 18. Handling times at the external terminal.

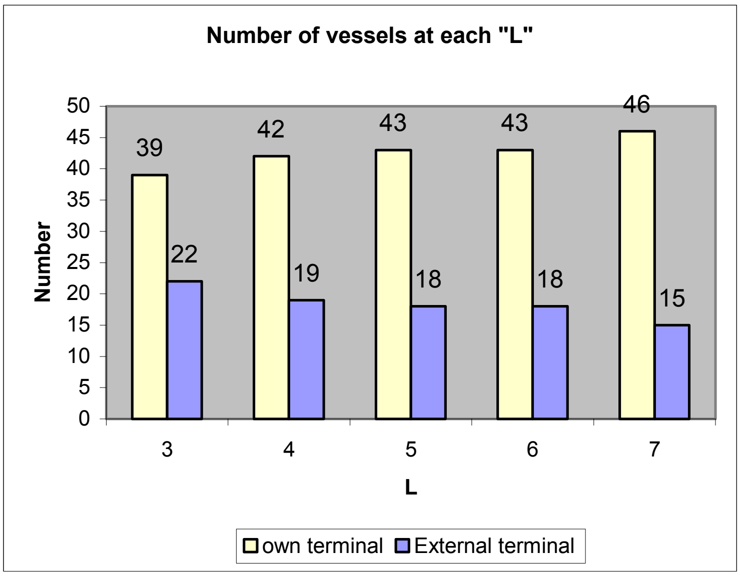

Figure 19 Number of vessels for different L's.

Figure 19 shows that with increasing L, the number of vessels served at the own terminal is increasing, while this trend is not significant from L = 4 to 6. To the contrary, more vessels are obviously directed to the external terminal with decreasing L.

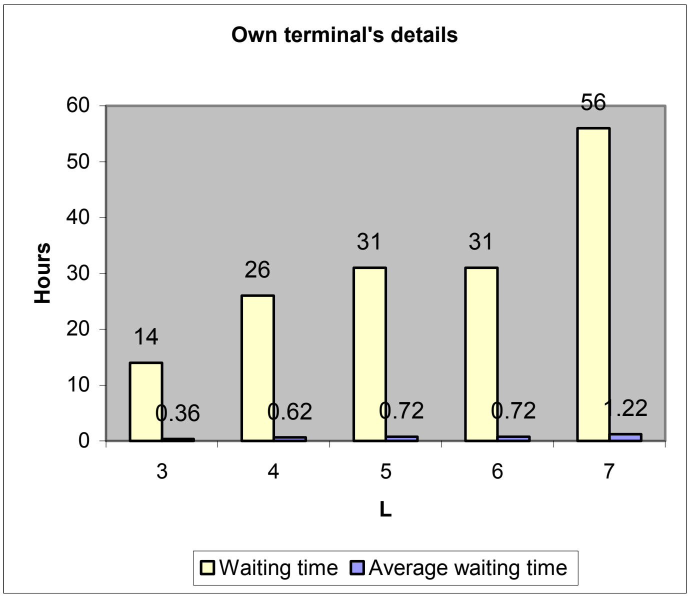

Figure 20. Waiting times and Average waiting times.

Table 6. Waiting times with different *L*'s.

L	**Total waiting time in hours**	**Average waiting time**	
		In hours	**In minutes**
3	14	0.36	21
4	26	0.62	37
5	31	0.72	43
6	31	0.72	43
7	56	1.22	73

Figure 20 and Table 6 reveal the waiting time in own terminal with varying L. When L is changed, waiting time is significantly changed. Furthermore, this significance occurred when increasing L from 3 to 5 and 6 to 7. When changing L from 3 to 4, 3 more vessels are served in own terminal, and from 6 to 7, another 3 more vessels are served in own terminal (from 39 to 42 and 43 to 46 respectively, from Figure 19) and the waiting time increases considerably from 14 to 26 hours and 31 to 56 hours accordingly (increase of 12 hours and 25 hours correspondingly from Figure 20). To the contrary, one includes one vessel in own terminal with L increased from 4 to 5 and no vessel with L increased from 5 to 6, respectively.

While the explicit objective is the minimization of the total handling time at the external terminal, the terminal operator implicitly aims at improvement of the customer satisfaction in terms of the waiting time of vessels and less costs in the operation associated with the external terminal in terms of the handling time of vessels. If the implicit objective is measured by the sum of the waiting time at own terminal and the handling time at the external terminal, the solutions with L of 4 to 7 are acceptable as shown in Table 7. However, the operator prefers the least value of the waiting time criterion in order to potentially force incoming vessels to wait for less time. From this point of view, the best is L of 4.

Table 7. The sum of handling and waiting times with different *L*'s.

L	**Handling time at the external terminal**	**Waiting time at own terminal**	**Sum of handling and waiting times**
3	239	14	253
4	210	26	236
5	202	31	233
6	202	31	233
7	180	56	236

CHAPTER SIX

FURTHER IMPROVEMENTS

In chapter three, the problem formulations of SEBAP and DEBAP were made. The relevant constraints were designed to frontier the course of the corresponding objective functions.

Some constraints may work toward easing the scope of the problem, while others may stand against the proceedings. In the latter case, one has to introduce new parameters in order to get away with the difficulties, which came across or modify the problems accordingly. Since the modification of a problem may lead to a total different scenario, it is always a realistic way to modify the constraints.

In this scope, one may look at the problem formulation in chapter three and put forward some modifications to some of the constraints in the following manner.

The methodology applied in this study is a simple heuristic based on a preliminary solution found by the classical Assignment Problem. It is likely that a better solution is obtained if a sub-gradient method with Lagrangian relaxation of the original problem is applied. In the following section, possible formulations by the Lagragian relaxation will be discussed.

6.1) SEBAP:

In order to apply the Lagrangian relaxation method for the SEBAP formulation (3-1) through (3-5), the constraint (3-4) should be relaxed. Due to constraint (3-4), the formulation is a non-linear integer problem, which is difficult to be solved. Instead of the time limit constraint, this constraint could be modified so that the average waiting time of ships scheduled to the own terminal would not exceed L. Then the constraint (3-4) can be re-written as follows:

$$\sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U}\left\{S_i - A_j + \sum_{l\in V}\sum_{m\in P_k} C_{il}x_{ilm}\right\}x_{ijk} \leq \sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U} Lx_{ijk} \tag{6-1}$$

Since the decision variables, x_{ijk} are restricted to the values 0 or 1 and also as the left hand side of the above inequality represents the total waiting time, this inequality can be formulated as follows, according to Imai et al. (2001) :

$$\sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U}\{(T-k)C_{ij} + S_i - A_j\}x_{ijk} \leq \sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U} Lx_{ijk} \tag{6-2}$$

Then, with the modification in the waiting time constraint, the SEBAP would take the following format:

Minimize $\quad H = \sum_{j\in V}\sum_{k\in U} C_{Qj}x_{Qjk} \qquad (6\text{-}3)$

subject to $\quad \sum_{i\in B}\sum_{k\in U} x_{ijk} = 1 \qquad \forall j \in V \qquad (6\text{-}4)$

$\sum_{j\in V} x_{ijk} \leq 1 \qquad \forall i \in B, k \in U \qquad (6\text{-}5)$

$\sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U}\{(T-k)C_{ij} + S_i - A_j\}x_{ijk} \leq \sum_{i(\neq Q)\in B}\sum_{j\in V}\sum_{k\in U} Lx_{ijk} \qquad (6\text{-}6)$

$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \qquad (6\text{-}7)$

Then, the following Lgrangian relaxation provides the lower bound of the formulation (6-3) to (6-7).

Minimize

$$H = \sum_{j \in V} \sum_{k \in U} C_{Qj} x_{Qjk} + \lambda \left[\sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} \{(T-k)C_{ij} + S_i - Aj\} x_{ijk} - \sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} L x_{ijk} \right] \quad (6\text{-}8)$$

subject to $\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V \qquad$ (6-9)

$\sum_{j \in V} x_{ijk} \leq 1 \qquad \forall i \in B, k \in U \qquad$ (6-10)

$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \qquad$ (6-11)

where λ is a Lagrange multiplier.

Note that the formulation (6-8) to (6-11) can be reformulated as follows:

Minimize $\sum_{i \in B} \sum_{j \in V} \sum_{k \in U} E_{ijk} x_{ijk} \qquad$ (6-12)

subject to $\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V \qquad$ (6-13)

$\sum_{j \in V} x_{ijk} \leq 1 \qquad \forall i \in B, k \in U \qquad$ (6-14)

$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U \qquad$ (6-15)

where E_{ijk} is a collective parameter to all relevant parameters in (6-8).

According to Imai et al. (2001), the formulation (6-12) to (6-15), which is a three dimensional Assignment Problem, can be reduced to a two dimensional Assignment Problem as shown below:

Objective function (6-12) could be reformulated by considering the following facts.

- Substitute for $i \in B$ and $k \in U$ by $n \in N$, where $|N|$ is the cardinality of set N and $|N| = |B| \times |U|$.

Then formulation (6-12)-(6-15) becomes,

$$\text{Minimize} \quad \sum_{j \in V} \sum_{n \in N} D_{jn} x_{jn} \tag{6-16}$$

$$\text{subject to} \quad \sum_{n \in N} x_{jn} = 1 \qquad \forall j \in V \tag{6-17}$$

$$x_{jn} \in \{0,1\} \qquad \forall j \in V,\ \forall n \in N \tag{6-18}$$

where D_{jn} is an equivalent parameter to E_{ijk}.

6.2) DEBAP:

The Lagrangian relaxation of the DEBAP formulation can be written, as in a similar manner to that of SEBAP, relaxing the constraint (3-9), which is:

$$\sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} \left\{ S_i - A_j + \sum_{l \in V} \sum_{m \in P_k} C_{il} x_{ilm} \right\} x_{ijk} + \sum_{i(\neq Q) \in B_i} \sum_{j \in V} \sum_{k \in U} \left\{ y_{ijk} + \sum_{l \in V} \sum_{m \in P_k} y_{ilm} \right\}$$
$$\leq \sum_{i \in B} \sum_{j \in V} \sum_{k \in U} L x_{ijk}$$

Constraint (3-9) is non linear; therefore as done for SEBAP, this constraint could be modified so that the average waiting time of ships scheduled to the own terminal would not exceed *L*. Then, Constraint (3-9) turns to be as follows:

$$\sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} \left\{ (T-k) C_{ij} + S_i - A_j \right\} x_{ijk} + \sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} (T-k) y_{ijk} \leq \sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} L$$

As for the modified problem, the Lagrangian relaxation would be:

Minimize $\sum_{j \in V} \sum_{k \in U} C_{Qj} x_{Qjk} + \lambda \sum_{i(\neq Q) \in B} \sum_{j \in V} \sum_{k \in U} \{(T-k)C_{ij} + S_i - A_j - L\} x_{ijk} +$

$$\lambda \sum_{i(\neq Q) \in B} \sum_{j \in W_i} \sum_{k \in U} (T-k) y_{ijk} \quad (6\text{-}19)$$

subject to

$$\sum_{i \in B} \sum_{k \in U} x_{ijk} = 1 \qquad \forall j \in V, \quad (6\text{-}20)$$

$$\sum_{j \in V} x_{ijk} \leq 1 \qquad \forall i \in B, k \in U, \quad (6\text{-}21)$$

$$\sum_{l \in V} \sum_{m \in P_k} (C_{il} x_{ilm} + y_{ilm}) + y_{ijk} - (A_j - S_i) x_{ijk} \geq 0$$

$$\forall i \in B, j \in W_i, k \in U, \quad (6\text{-}22)$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in B, j \in V, k \in U, \quad (6\text{-}23)$$

$$y_{ijk} \geq 0 \qquad \forall i \in B, j \in V, k \in U, \quad (6\text{-}24)$$

Unfortunately, this relaxation problem is not easy to solve. If one pursues applying the sub-gradient method by using the Lagrangian relaxation, one can use the optimal solution to the problem, which is relaxed with constraint (6-22).

6.3) Physical restrictions:

When physical factors are to be included in the improvement of the problem, it is possible to consider,

- Lengths of the berths
- Depths of the berths
- Lengths of the vessels
- Drafts of the vessels etc., in addition to the constraints formulated in the original problem.

These restrictions would add more constraints to the original problem and therefore relevant modifications are necessary in the problem formulation.

6.4) Service priorities:

Further, various priorities involved in serving vessels may also be taken into account. This may be done according to the typical division of mother/main liner vessels and feeder vessels in shipping.

6.5) Algorithm:

In chapter four, the solution algorithm does not reallocate the vessels, in a particular berth, when there is a sufficiently large idle time in a previous location of that berth assignment, once the selection of a candidature from the pool is underway. This methodology was adopted because of the need to have a simplification in the programme developed. Therefore one can promote this methodology by means of introducing a process, which addresses the above-mentioned difficulty.

CHAPTER SEVEN

SUMMARY AND CONCLUSIONS

This study focuses on an efficient scheduling method, which may be applicable to berth allocation to calling vessels at the port of Colombo, Sri Lanka where there are two container terminals operating opposite each other. For various reasons, some vessels calling at one of the terminals called Jaya Container Terminal (JCT) have to be moved to the external terminal facility known as South Asia Gateway Terminal (SAGT), whenever there is a congested situation at JCT. Vessels are moved depending on their predictable waiting times for berths at JCT.

This dissertation aims at finding a methodology to develop a computer-based programme that may produce the optimum results in order to minimise the number of vessels allocated to the external terminal with resulting huge costs born by JCT in such an operation. The major findings are described as follows:

In chapter one, a comprehensive treatise about the background of the identified problem is given. It further discusses the costs related with the existing practices, with regard to the congested situations at JCT and the need to minimise those costs, and if possible, to eliminate these completely.

Towards this aspiration, many works of different authors needed to be investigated, in order to take a correct approach to the intended problem formulation. There is a great deal of research pursued by various authors regarding somewhat similar problems. Chapter two covers the related literature, regarding the work of the dissertation.

Chapter three makes the problem formulation with the static and dynamic situation of incoming vessels to be served. The problem concerned could be extended to some forms, in order to facilitate the solution procedure and/or in order to have another objective. Modifications of the objective and/or constraints may lead to such extensions to the original problem. As they will be shown in one of the subsequent chapters, chapter three focuses on the original version of the objective, by employing the necessary conditions in their original form.

Chapter four demonstrates the outline of a solution algorithm. The algorithm for two types of the problem, namely SEBAP and DEBAP is comprehensively discussed and the functional configuration of the algorithm is demonstrated through computational examples.

Chapter five consists of the computational experiments performed using an actual set of data obtained from JCT in which there are vessels to be directed to the external terminal. In addition to this, computer programmes developed not only for main computation to determine berth allocation but also for some auxiliary computational tasks, are explained in this chapter together with the data sets employed. Furthermore, intensive discussions on the computational results are made. In real application, it is often that the entire scheduling is split into some small horizons especially when the entire problem is considerably huge. In order to examine the relationship between the solution quality and the number of splits, various experiments with different numbers of splits are performed. As a results, it is found that a computation with six splits, each having 10 vessels produce the preferable quality of the solution, taking into account the uncertainty involved with ongoing updates of arriving ships during planning horizon in a split. Also, the experiments with various stipulated waiting time are executed to identify the practical value of the waiting time to balance workloads between own terminal and external terminal.

All the foreseeable extensions to the original problem formulation in terms of easiness of solving problems, quality of the solution, and applicability to complex real situations are demonstrated in chapter six. These developments are mainly centred with the modifications to the constraints whereas physical restrictions are also looked into in this chapter.

As already stated, according to the computational experiments, the berth allocation planning split into 6 phases with each split having ten vessels would be the most preferable. According to the experiments with different waiting time stipulated, *L*, when increasing *L* from 4 to 6 hours, only one vessel has been suffering from a waiting time. Once reducing *L* from 6 to 3 hours, more vessels have been suffering from a waiting time range between 3 and 4 hours than the one between 4 to 6 hours. In reverse interpretation, when *L* is decreased from 7 to 6 hours, there is a tremendous improvement in the waiting time of vessel served in the own terminal. This improvement is traded off with increasing handling time at the external terminal.

All in all, solutions with *L* ranging from 4 to 7 hours are acceptable. However, in fine-tuning of *L*, precautions have to be taken in order to make sure that only a least possible number of vessels have to be directed to the external terminal. This suggests *L* of 4 hours.

Also *L*, when increasing, has to be more than or equal to a vessel's handling time, which determines a successive vessel to be included in the pool, in a particular berth. Increasing *L* could be done bearing in mind the average waiting time for a vessel.

In conclusion, taking into account the whole discussion, it can be stated that the optimal split of vessels would be 10 vessels together with $L = 4$ having reached the goal of minimising costs associated with diverting vessels, to a minimum possible extent, and keeping intact the customer satisfaction to a maximum possible altitude.

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